Impedance Parameter Theory

I. Definition of Impedance Parameters

Impedance is an important parameter used to characterize electronic circuit, component, and the materials used to make components.

Impedance (Z) is generally defined as the total opposition a device or circuit offers to the flow of an alternating current (AC) at a given frequency. Impedance is a concept describing AC signal, and DC signal is only described by resistance. Resistance in DC circuit (R Dc), is sometimes directly written as DCR to distinguish with resistance in AC circuit.

Impedance is represented as a complex quantity which is only completely shown on a vector plane, so it's sometimes called complex impedance. Complex impedance can be expressed using either the rectangular-coordinate form R+jX or in the polar form as a magnitude and phase angle: $|Z|\angle\theta$. And the parameters in the two polar forms can be converted through the mathematical relationship shown in Figure 1.

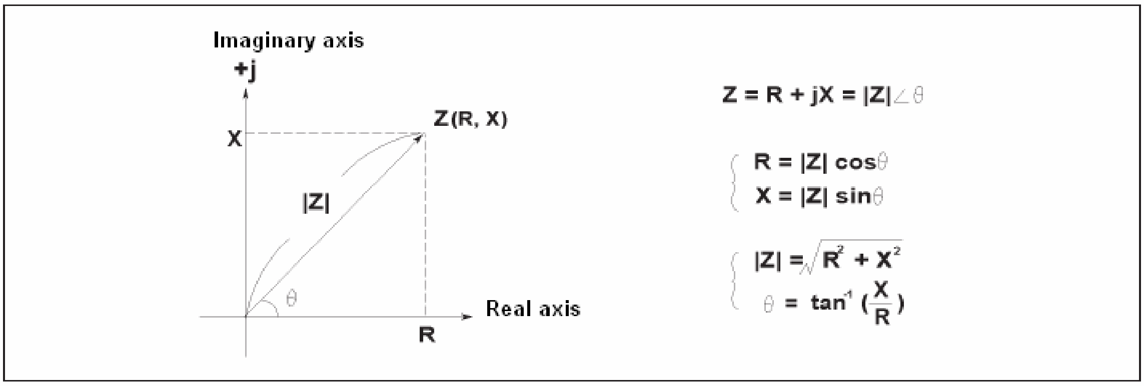

Figure 1 Impedance (Z) consists of a real part (R) and an imaginary part (X)

Figure 1 shows the vector plane of complex impedance, which consists of a real part (resistance R) and an imaginary part (reactance X). The absolute value of impedance $|Z|$ and the angle θ between impedance and abscissa. And Figure 1 also shows the mathematical relationship between R, X, $|Z|$ and θ. The formula is as follows: $Z = R + jX$, $Z = |Z|e^{j\theta}$.

Resistance (R) is a real part of impedance (Z) and it serves to transfer power to heat energy in the circuit. Reactance (X) is an imaginary part of impedance (Z), and it serves to store energy in the circuit. $|Z|$ is the absolute value of complex impedance, and θ is the angle between Z and real axis. And their relationship is as follows: $|Z| = \sqrt{R^2 + X^2}$, $\theta = arctg\left(\frac{X}{R}\right)$------------------ (1)

$$R = |Z|\cos\theta, \quad X = |Z|\sin\theta \text{ ---------------------- (2)}$$

In some cases, using the reciprocal of impedance is mathematical expedient. In which case 1/Z=1/(R+jX)=Y=G+jB, where Y represents admittance, G conductance, and B susceptance, as shown in Figure 2. It can be physically defined as the opposition a device or circuit offers to the flow of an AC at a given frequency.

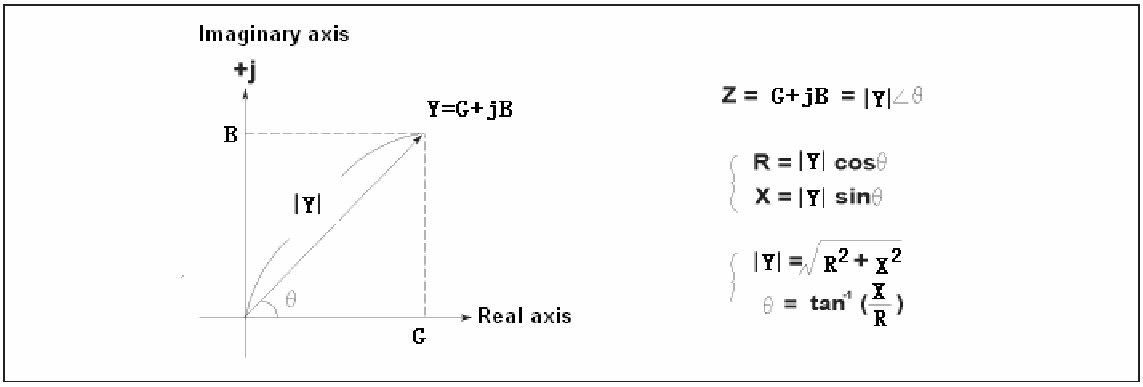

Figure 2 Admittance (Y) consists of a real part (conductance G) and an imaginary part (susceptance B)

And with the same theory, their relationship can be expressed as follows:

$$|Y| = \sqrt{G^2 + B^2}, \quad \theta = arctg\left(\frac{B}{G}\right) \text{---------------(3)}$$

$$G = |Y|\cos\theta, \quad B = |Y|\sin\theta \text{ --------------------(4)}$$

The unit of impedance, resistance and reactance is the ohm (Ω), and admittance, conductance and susceptance is the siemen (S). Impedance is a commonly used parameter and is especially useful for representing a series connection of resistance and reactance, because it can be expressed simply as a sum, R and X. For a parallel connection, it is better to use admittance (see Figure 2).

II. Component Parameters Used and Amplified in Measurement

In real world, three basic passive components, resistor, inductor and capacitor, are usually used. Theoretically speaking, all the passive components can be described with these three components or their combination, such as piezoelectric component, crystal oscillator and, semiconductor. They make up the whole parts of electric circuit with basic active components, like diode, rectifier, triode, field-effect-transistor. Therefore, it's very important to know passive component's characteristics well and truly.

In natural world, there are no pure resistance, inductance, and capacitance exiting. Any component is combination of some several components, and the combination from is dependant on frequency. For example, a capacitor has the following parasitic parameters: down-lead resistance, down-lead inductance, insulation resistance, etc.

For the sake of succinctness, a component's parameter combination form is generally described as equivalent series connection and equivalent parallel connection, which can describe form of a basic component (L, C, R) at low-frequency area (≤2MHz).

Equivalent series connection means series connection of a resistance Rs (conductance Gs, Rs=1/Gs) and reactance Xs (susceptance Bs, Xs=1/Bs). Subscript s means series connection.

Equivalent parallel connection means parallel connection of a resistance Rp (conductance Gp) and reactance Xp (susceptance Bp). Subscript p means parallel connection.

The two equivalent circuit connections are shown in Figure 3.

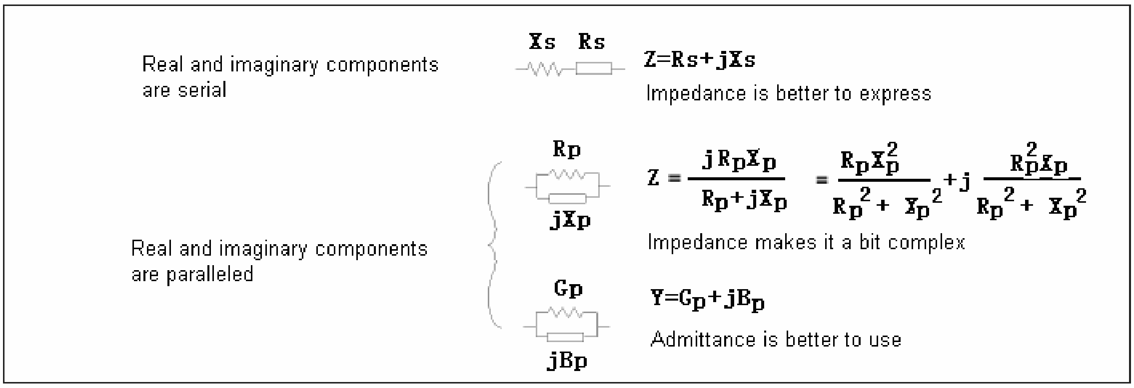

Figure 3 Expression of series and parallel combination of real and imaginary components

Generally, equivalent series connection is expressed through resistance Rs and reactance Xs (Z=Rs+jXs), while parallel connection through conductance Gp and susceptance Bp (Y=Gp+jBp).

And the two equivalent forms can be converted (the two forms shown in figure 3), and their conversion relationship can be described as follows:

$$R_s = \frac{R_p X_p^2}{R_p^2 + X_p^2},\quad X_s = \frac{R_p^2 X_p}{R_p^2 + X_p^2} \text{ --- } (5)$$

And we can see $\frac{R_s}{X_s} = \frac{X_p}{R_p}$ through formula (5), which is defined as D. D's physical meaning is expressed in the following text, and it means dissipation factor D, then formula (5) can also be expressed as:

$$R_s = \frac{R_p}{1+ {}^{1}\!/_{D^2}}\,,\quad X_s = \frac{X_p}{1+D^2} \quad \text{--}\ (6)$$

We can infer formula (7) from formula (6):

$$R_p = (1+ {}^{1}\!/_{D^2})R_s\,,\quad X_p = (1+D^2)X_s \quad \text{---}\ (7)$$

And conversion relationship of G and B between the two equivalent modes can be inferred according to the same principle.

Reactance X is energy stored part of impedance Z. Two components can store energy, inductance L and capacitance C, so reactance X takes two forms: inductive (XL) and capacitive (XC). By definition, XL=2πfL, and XC=1/ (2πfC), where f is the frequency of interest, L is inductance, and C is capacitance. And 2πf can be substituted for by the angular frequency (ω: omega) to represent XL=ωL, XC=1/ (ωC). Refer to Figure 4.

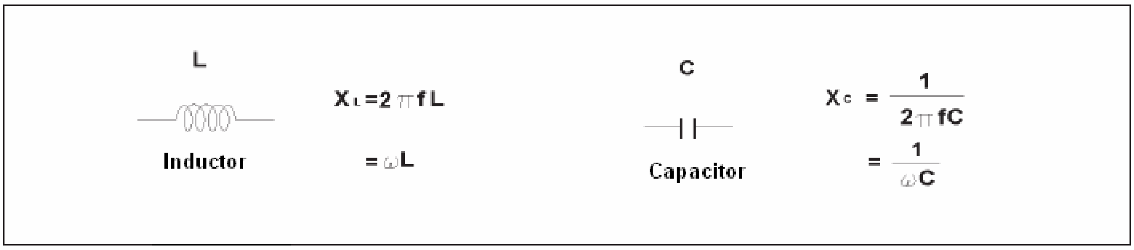

Figure 4 Reactance in two forms ---- inductive (X_L) and capacitive (X_C)

A similar reciprocal relationship applies to susceptance and admittance. Figure 5 shows a typical representation for a resistance and reactance connected in series or in parallel.

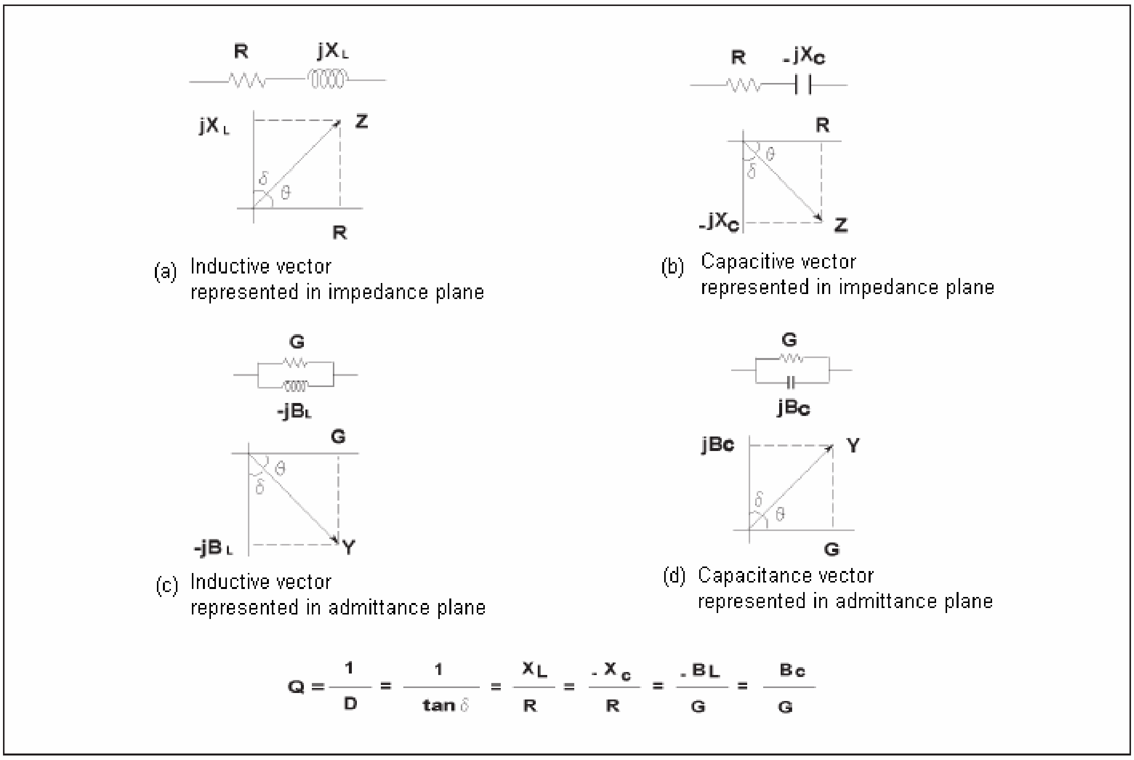

Figure 5 Relationships between impedance and admittance parameters

Complex impedance Z has energy dissipated part R and energy stored part X. Therefore, the ratio of dissipated energy and stored energy is dissipation factor S.

In series connection mode, $D=\frac{P_d}{Ps}=\frac{I^2R_s}{I^2X_s}=\frac{R_s}{1/\omega C_s}=\omega R_s C_s$ --------------------------------- （8）

In parallel connection mode, $D=\frac{P_d}{Ps}=\frac{U^2/R_p}{U^2/X_p}=\frac{X_p}{R_p}=\frac{1}{\omega R_p C_p}$ --------------------------------- （9）

S is always the same whether in series connection mode or parallel connection mode.

The quality factor Q serves as a measure of a reactance’s purity (how does it is to being a pure reactance, no resistance), and is defined as the ratio of the energy stored in a component to the energy dissipated by the component. Q is expressed as Q=1/D.

From Figure 5, we can see that Q is the tangent of the angle θ. For capacitors the term more often used to express purity is dissipation factor (D). This quantity is simply the reciprocal of Q, that is the tangent of the complementary angle of θ, the angle δ shown in Figure 5 (d).

III. Unit and Dimension of Impedance Parameter

No.	Symbol	Name	Unit	Dimension

1	$\|Z\|$	absolute value of impedance	ohm	μΩ， mΩ, Ω, kΩ, MΩ
2	R	resistance		
3	X	reactance		
4	$\|Y\|$	admittance	siemen	μS, mS, S, kS
5	G	conductance		
6	B	susceptance		
7	θ	phase angle	angle DEG radian RAD	1°（DEG）=180° /π×RAD
8	C	capacitance	farad	pF, nF, μF, mF, F
9	L	inductance	henry	nH, μH, mH, H
10	D	dissipation factor	none	none
11	Q	quality factor		

$p=10^{-12}$, $n=10^{-9}$, $\mu=10^{-6}$, $m=10^{-3}$, $k=10^{3}$, $M=10^{6}$

Changzhou Tonghui Electronics Co., Ltd.

Add: No.3 Tianshan Rd., New & Hi-tech District, Changzhou City, Jiangsu Province, P.R. China
Tel: 00-86-519-5132222, 5115008, 5109671, 5113342, 5109592
Fax: 00-86-519-5109972, 5195190
Postcode: 213022
Website: www.tonghui.com.cn
E-mail: Sales Department sales@tonghui.com.cn
R&D Department gzq@tonghui.com.cn